ÉCOLE CENTRALE LYON

membre de : UNIVERSITÉ DE LYON

3e année
Année 2017-2018

Titre	Responsables	Charge Planifiée
Physical problems in unbounded media : mathematical analysis and numerics	**Laurent Seppecher** Grégory Vial Abdelmalek Zine	**10H CM** **6H TD** **12H BE**

Goals

This course aims at giving the mathematical foundations for the study of partial differential equations posed in an unbounded domain. We will focus on model equations (Laplace, Helmholtz, wave equation) to present the mathematical framework and the main ideas for the design of numerical methods.

Keywords :

Propagation phenomena. Partial differential equations. Unbounded domains.

Skills :

To be able to identify conditions for closing a problem in an unbounded domain.
To be able to design a numerical method for PDEs in unbounded domains.
To be able to quantify the accuracy of such a numerical method.

Program :

Part I : **Basic facts for stationary and harmonic problems**

Functional framework : Sobolev spaces, Fourier analysis.
Asymptotic analysis at infinity for the Laplace operator.
Application to the study of harmonic guided waves (scattering boundary conditions, perfectly matched layers).

Part II : **Time dependent problems**

The wave equations in the free space : D'Alembert formula. Resolution using Fourier analysis.
Absorbing boundary conditions and pefectly matched layers.

Numerical schemes for the wave equation.

Part III : **Focus on the Helmholtz problem in the free space**

Existence and uniqueness : Sommerfeld radiation condition.
Fundamental solutions, Green functions. Integral representations.
Fast numerical methods, fast convolution.
Scattering problems : wave interactions with inhomogeneous media, Born approximation, Born series.
Application in inverse problems : back-scattering, application to sonar/radar detection and ultrasound imaging.

Options/Masters :

Option MD-MIR.

Masters : Mathématiques appliquées, statistique. Aerospace engineering. Acoustics.

Bibliography :

J.-C. Nédelec, *Acoustic and Electromagnetic Equations*, Springer, 2001.

Dan Givoli, *Numerical Methods for Problems in Infinite Domains*, Studies in Applied Mechanics 33, Elsevier 1992.

Lutz Lehmann, *Wave Propagation in Infinite Domains*, Lecture Notes in Applied and Computational Mechanics 31, Springer 2007.

Evaluation

Simulation project (50%) and Final exam (50%).