Open position for an internship at *POEMS*

UMR 7231 CNRS/INRIA/ENSTA

2012/2013 Academic year

A Calderon preconditioner for the Fast Multipole Accelerated Boundary Element Method in 3D visco-elastodynamics

- **Scientific Context.** The modeling of seismic wave propagation to understand complex phenomena such as site-effects or soil-structure interaction is an active area of research. The difficulties are related to the complexity of the system to model and the large spatial scale of the problems. Currently, 3D simulations are still limited to simplified configurations. This internship is part of an effort towards the development of tools to simulate real-life problems related to seismic wave propagation.

  Various numerical methods can be used to simulate seismic wave propagation. The main advantages of the Boundary Element Method (BEM) are to reduce the discretization to the domain boundary and to exactly take into account radiation conditions at infinite. As a result, BEMs are well suited to deal with problems in (semi-)infinite domains. However standard BEMs lead to a fully-populated influence matrix, and are thus severally limited regarding problems with complex geometries or in a large frequency range [3]. To address these limitations, recent works, in our group, have concerned the Fast Multipole accelerated BEM for 3D visco-elastodynamics [1, 4]. The Fast Multipole Method (FMM) permits to reduce drastically the solution time and the memory requirements of the BEM and to considerably enlarge the BEM model size.

  The solution of the BEM system is done with an iterative solver (i.e. GMRES) of complexity $O(n_{iter}\mu(N))$ in terms of CPU time (where $\mu(N)$ represents the cost of a matrix-vector product for N degrees of freedom (DOF) and n_{iter} is the number of iterations of the iterative solver). The FMM reduces the cost of the evaluation of the linear integral operator, main step of the computation of a matrix-vector product required by the iterative solver. In 3D elastodynamics, μ has been shown to be $\mu(N) = N \log N$. However, the solution time depends also on the number of iterations. If no preconditioner is used this number (n_{iter}) may be large and may also increase with N and with the frequency.

- **Objectives.** Since, in the context of the FMM, the influence matrix is never explicitly assembled in totality, the definition of an efficient preconditioner is not an easy task. Algebraic preconditioners (SPAI, ILU, diagonal, ...) have been adapted to this constraint but have been shown to have real but limited efficiency [2]. A possible explanation is that they are based on the discretized system. Another approach consists in formulating a preconditioner based on the mathematical properties of the continuous integral operators. Towards this goal, Calderon relations can be considered.

The proposed work is decomposed into the following three steps:

(i) Formulation of a preconditioner for 3D elastodynamics FM-BEM (based on the full-space fundamental solution).

(ii) Implementation of the proposed preconditioner into the code COFFEE (solver for 3D elastodynamic problems based on the FM-BEM).

(iii) Numerical experimentations to assess the efficiency, and in particular to verify whether the number of iterations is independent of the frequency and of N.

▷ **Knowledge.** Solid backgrounds in applied mathematics or computational mechanics and scientific programming skills are expected. Prior knowledge on the boundary element methods will be appreciated. Scientific interest in Earth problems is an asset.

▷ **Contact and location.** The internship (expected duration 5-6 months) will take place in the POems team (Propagation d'ondes: études mathématiques et simulation) of the applied mathematics department at ENSTA (Palaiseau). It will be supervised by Stéphanie Chaillat (CNRS junior scientist) and Marc Bonnet (CNRS senor scientist). Applications including a CV and a cover letter have to be sent to stephanie.chaillat@ensta.fr.

▷ **Prospect.** Depending on progress and results, this internship work may result in a published peer-reviewed article. Moreover a PhD continuation on this topic is a possibility.

The position is open to an international student.

References

[1] S. Chaillat, M. Bonnet, J.F. Semblat, A multi-level Fast Multipole BEM for 3-D elastodynamics in the frequency domain. *Computer Methods in Applied Mechanics and Engineering*, 197: 4233-4249, 2008.

[2] S. Chaillat, J.F. Semblat, M. Bonnet, A preconditioned 3-D multi-region fast multipole solver for seismic wave propagation in complex geometries. *Communications in Computational Physics (special issue WAVES 2009)*, Vol. 11, 594-609, 2012.

[3] N. Delepine, J.F. Semblat, Site effects in an alpine valley with strong velocity gradient: Interest and limitations of the 'classical' BEM. *Soil Dynamics and Earthquake Engineering*, Vol. 38, 15-24, 2012.

[4] E. Grasso, S.Chaillat, M.Bonnet, J.-F. Semblat, Application of the multi-level time-harmonic fast multipole BEM to 3-D visco-elastodynamics. *Engineering Analysis with Boundary Elements*, Vol. 36, 744-758, 2012.