



## Internship for Master 2 in Applied Mathematics

# Parallel Krylov solvers applied to multi-trace integral formulations of wave propagation

**Context** Derivation of efficient Domain Decomposition Methods (DDM) for the simulation of high or even moderate frequency wave propagation phenomena is nowadays a numerical challenge. In the case of piecewise homogeneous propagation media, a natural idea consists in proposing a DDM built on top of a boundary integral formulation of the problem.

Boundary integral formulations consist in rewriting (a priori) volumic PDEs as equivalent equations posed only on the boundary of the computational domain. Although more complicated than volumic formulations of PDEs, integral formulations give rise to much less dispersive numerical methods. But still very little is known about DDM involving boundary integral operators.

Boundary integral equation techniques are of particular interest for simulation in many applicative contexts involving electromagnetic wave propagation in harmonic regime, covering applications such as cell-phones, airborne electronics, antenna design, wave guides. It is of particular relevance in highly oscillatory problems due to their robustness with respect to numerical dispersion effects.

This internship will be part of the project NonLocalDD<sup> $\dagger$ </sup> funded by the french National Research Agency (ANR).

**Objectives** The candidate will focus on a particular type of boundary integral methods known as Multi-Trace Formulations (MTF) introduced very recently, see [1, 2]. These formulations are taylored for problems where the geometry of the computational domain involves many subdomains each one of which is associated to different constant material characteristics.

The internship will start with a bibliographical work on boundary integral formulations of harmonic wave propagation and domain decomposition. Then, considering problems of wave scattering by composite objects, the candidate will conduct numerical experiments so as to examine the performance of various usual global iterative solvers: block Jacobi, block Gauss-Seidel and GMRes when applied to a multi-trace formulation of the problem. Such investigations will rely on implementations of MTF in adequate situations, making use of an already implemented C++ library.

Candidate profile The candidate should have a solid background on the numerical analysis of elliptic PDEs (variational theory of the laplacian, Lax-Milgram theorem, finite element method) and some programming experience, ideally in C++.

**Possible continuation as a Ph.D. thesis** A candidate providing satisfactory commitment for this internship may be offered a PhD fellowship funded by the French National Research Agency.

 $<sup>^{\</sup>dagger} https://www.ljll.math.upmc.fr/{\sim}claeys/nonlocaldd/index.html$ 

### **Practical information**

Location: Jacques-Louis Lions Laboratory (LJLL), Univ. Paris 6, Jussieu. Duration: 5 month.

#### Contact:

Xavier Claeys (LJLL, INRIA Alpines) Email: claeys@ann.jussieu.fr Web: https://www.ljll.math.upmc.fr/~claeys/

Frédéric Nataf (LJLL, INRIA Alpines) Email: nataf@ann.jussieu.fr Web: http://www.ann.jussieu.fr/nataf/

### References

- [1] R. Hiptmair & C. Jerez-Hanckes, Multiple traces boundary integral formulation for Helmholtz transmission problems, Adv. Comput. Math., 37(1):39-91, 2012.
- [2] X. Claeys, R. Hiptmair & C. Jerez-Hanckes, Multi-trace boundary integral equations, chapter in Direct and Inverse Problems in Wave Propagation and Applications, 51âĂŞ100, Radon Ser. Comput. Appl. Math., 14, De Gruyter, Berlin, 2013.