Internship 2019: Structure of rational matrices in connection with scattering theory in electronics.

Expected duration: *4-6 months (with regular remuneration).*
Level: *Second year of Master degree or Engineers School (PFE).*
Location: *INRIA Sophia Antipolis, BP 93, 06902 Sophia-Antipolis Cedex, France.*
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Context.
Passive microwave systems are usually modeled by scattering matrices reflecting the physical properties of the device. In a finite dimensional context, these matrices are rational and their McMillan degree corresponds to the number of state variable. Stability of the system implies analyticity in the right half-plane, so that the poles of the matrix lies in the left half-plane. For a conservative device, the evaluation of the matrix at any imaginary point is unitary. Such a stable rational matrix is called LQMD lossless LQLJ referring to the fact that energy is neither created nor dissipated in the underlying system. In addition, electrical networks possess a reciprocity property which translates into symmetry of the scattering matrix, a far reaching property for rational matrices.
In practice, devices are measured thanks to high precision measurement instruments (Frequency Response Analysers). Measurements can be viewed as evaluations of the rational transfers at some frequency points in an interval of the imaginary axis. We are interested in a common situation where only a row of the scattering matrix is measured. The inverse problem we wish to address is then the following: given measurements of a row of the scattering matrix of an electronic device, is it possible to compute an approximation of the whole matrix? Applications include new challenging topics for the team, such as the recovery of metallic forms, typically a sphere, via inverse scattering methods, or else, the construction of a reliable scattering mathematical model for antennas.

Goals.
The internship mainly focus on the mathematical study of lossless symmetric matrices and extension issues : given a 1 x p row vector of rational functions, is it possible to extend it into a p x p lossless symmetric matrix? Under which conditions? Is the solution unique? Can it be done without increasing the complexity of the model (degree)? To answer these questions, a bibliographical study will be made, including [1] and [2] which deal with a less constrained but closely related extension problem. Then, a rational approximation method to approach the practical inverse problem will be investigated, based on previous work and on existing codes developed by Factas. The method could then be tested against simulated or measured data to find a mathematical model for some devices of theoretical or practical interest.

Required skills.
The internship topic requires skills in mathematics and modelization, together with a taste for practical applications. A standard knowledge in function theory, complex analysis, linear algebra, and system

theory will be useful. The internship may be followed by a PhD thesis.

References:

[1] *Minimal symmetric Darlington synthesis*,
L. Baratchart, P. Enqvist, A. Gombani, M. Olivi,
Math. of Control, Signal & Systems 4(2007), 283-311

[2] *Polynomial structure of $3\times$ 3 reciprocal inner matrices*,
D. Avanessoff, M. Olivi and F. Seyfert,
Proceedings of the MTNS 2010 (Budapest, Hungary)