Internship, 5-6 months, 2021.

Inverse problems in paleomagnetism magnetic moment estimation.

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We are interested with the following problem arising in paleomagnetism: from planar incomplete measurements of the magnetic field taken above a rock sample, we aim at recovering the magnetization distribution inside the rock. This is a particular instance of a wider class of problems: inverse potential problems, where the source term is to be recovered from measurements of the induced field taken at some distance from its support. This kind of problems plays an important role in many scientific domains.

The Factas team collaborates for several years with the Earth Sciences Department of MIT on inverse magnetization problems on rocks. The overall goal is to help geologists to study the history of the Earth magnetic field and other celestial bodies. The MIT's laboratory has an instrument, called SQUID microscope, that can measure very weak magnetic fields. It is used to get a map of the vertical component of the field on discrete points of a rectangle above a rock sample. From these measurements, geologists would like to recover the pointwise magnetization inside the rock.

This inverse problem is ill-posed. In particular, it suffers from non-uniqueness, *i.e.* there exist so-called silent sources: non-trivial magnetizations that emit no field outside the sample and are hence undetectable. However, it is in principle possible to recover the net moment (mean value) of a magnetization from the available data, because silent sources are proved to admit a net moment equal to zero.

In order to solve this moment estimation problem, we proposed a linear estimator technique: we compute functions f such that their scalar product against the field data are as close as possible to the net moment of the magnetization, under some regularization constraint. The overall goal of this internship is to improve the computation of f.

Such functions f can be obtained as solutions to some best constrained approximation problem (or bounded extremal problem, BEP), see our article [1]. The algorithm proposed in the above article in order to compute f suffers from several limitations: it is time expensive, which makes it nearly impossible to use for real situations; it is designed to recover the net moment of a 2D planar sample but must still be extended to 3D samples; finally, it approximates f by an element of a finite dimension space without granting any control on the quality of the final approximation with respect to the true function f.

The internship will start by a familiarization with the problem and the computational and mathematical tools used to solve it. A first work will consist in implementing the current algorithm (for which the current implementation should only be considered as a proof of concept). For this purpose, accurate explicit formulas recently obtained can prove useful. In a second time, an alternative algorithm to compute the same solution of the BEP, but with a completely different approach (using a basis of eigenfunctions of the Laplacian operator), will be investigated, in order to see how it compares with the first one. In both cases, the goal is to really rigorously compute an approximation of the true function f, with a controlled approximation accuracy, and with efficiency in mind.

If time permits, the formulation of the BEP will then be extended to compute linear estimators for 3D samples and it will be investigated how the previous algorithms can be updated to compute the solution of the 3D problem. A longer term goal is to design a method theoretically less accurate than solving the BEP to the benefit of being computationally more practical, and yet accurate enough to reliably estimate the net moment of real samples. If the candidate is good and motivated enough during the internship, a PhD could follow as a continuation of the exploration of these subjects.

The internship will take place in the Inria research center of Sophia Antipolis near Antibes in the south of France. The candidate must hold a Master 2 in applied mathematics or computer science and have programming skills (knowledge of Matlab is a plus but not mandatory).

References

[1] L. Baratchart, S. Chevillard, D. P. Hardin, J. Leblond, E. A. Lima, J.-P. Marmorat, Magnetic moment estimation and bounded extremal problems, Inverse Problems and Imaging, 13 (1), 2019, http://hal.archives-ouvertes.fr/hal-01623991.