Optimization and learning approach for model identification in quasi-static elastography imaging

Matser 2 internship proposal

Contacts

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Scientific context. Local values of the elastic parameters can be used as a discriminatory criterion to differentiate healthy from diseased tissues (in particular malignant tumors, even at early stages) [5]. Elastography is currently widely used in the diagnosis of common affections such as breast cancer or liver fibrosis. The measure of the stiffness of the liver is the reference non-invasive diagnostic method [2]. At the micrometer scale, it is now clear that the mechanical properties of cells and extracellular matrix are key factors of physiology and pathophysiology of biological tissues [4, 7]. Investigating these properties is of major interest both in fundamental biology and in medicine,)n particular in the context of breast tumor detection.

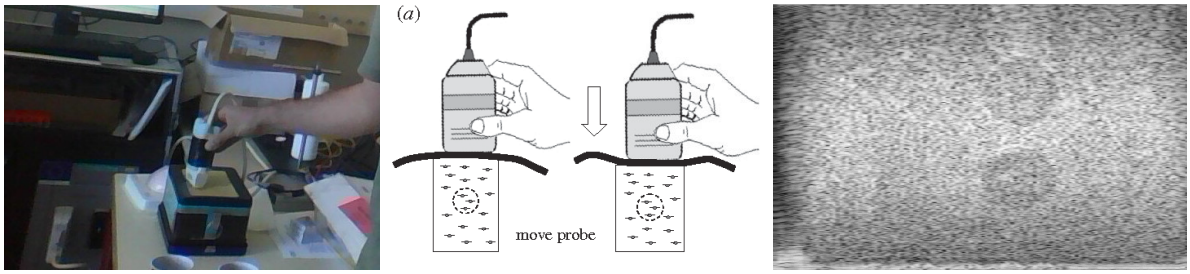

Figure 1: Acoustic data acquisition process for quasi-static elastogrpahy

It is possible, using various techniques (acoustic for instance), to measure displacement fields inside a living tissue. The general inverse problem of the quasi-static elastography imaging is to recover the elastic properties of the material $\boldsymbol{C}$ from one or several measured displacement fields $\boldsymbol{u}$ satisfying the elastic equilibrium equation

$$\mathbf{div}(\boldsymbol{C} : \mathcal{E}(\boldsymbol{u})) = \mathbf{0} \qquad \text{in } \Omega. \tag{1}$$

where $\mathcal{E}(\boldsymbol{u})$ is the strain matrix. Under some model simplifications problem can be reduced to find the shear modulus μ satisfying

$$\mathbf{div}(\mu \boldsymbol{T} : \mathcal{E}(\boldsymbol{u})) = \mathbf{0} \qquad \text{in } \Omega. \tag{2}$$

where $\boldsymbol{T}$ is a fixed four order tensor which depends on the elastic model assumptions. If $\boldsymbol{T}$ is known, a Galerkin approach can be efficient to recover μ. Choosing a pair a finite element spaces $M_h \subset L^2(\Omega)$ and $V_h \subset H_0^1(\Omega)^3$, the discrete problem can be solved as finding the first singular vector of the matrix

$$A_{ij} := \int_\Omega \varepsilon_j \mathcal{E}(\boldsymbol{u}) : \boldsymbol{T} : \mathcal{E}(\boldsymbol{e}_i) \tag{3}$$

in some sense, where (ε_j) and $(\boldsymbol{e}_i)$ are the bases of M_h and V_h respectively.

In [1], we proved the theoretical feasibility of solving this problem, and in [3, 6], in collaboration with the team of E. Brusseau (CREATIS), we applied this method with success on experimental and clinical data obtained from quasi-static ultrasound-elastography.

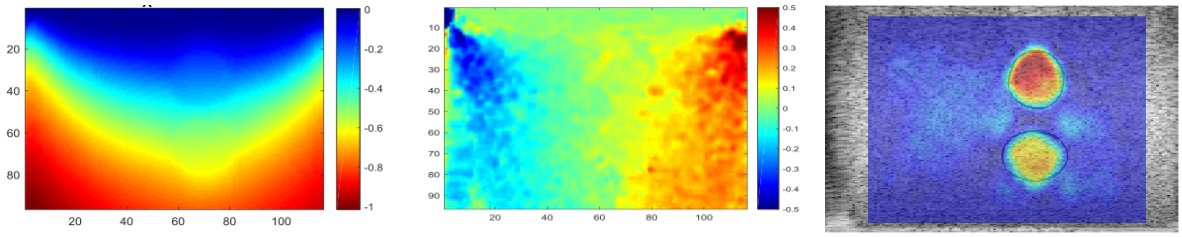

Figure 2: Measured displacement fields $\boldsymbol{u}$ and reconstructed shear modulus μ.

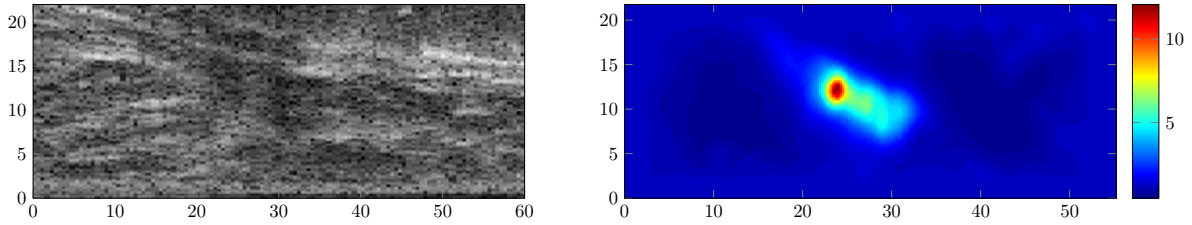

Figure 3: Reconstruction of the shear modulus of *in-vivo* malignant breast tumor from quasi-static elastography (data from E. Brusseau - INSA/CREATIS) $h = 0.7$ mm.

Objectives. So far, the simplified model based on a plane stress approximation (which allows to reduce a 3D problem to a 2D problem) is used and conduces to a known fixed tensor $\boldsymbol{T} = \boldsymbol{I} + I \otimes I$. This approximation is known to be false as soon as the displacement is not asymptotically small. This leads to an important error in the contrast estimation in the shear modulus reconstruction.

The first objective of this internship is to propose and test some improvements of this approach considering that the tensor $\boldsymbol{T}$ is unknown. It shall be estimated from numerical 3D and experimental data using an optimization approach or a deep learning strategy. The proposed model will be tested on simulated 3D data to recover the unknown parameter μ. Results will be compared to state-of-the-art methods for elastography imaging.

A secondary objective is to make use of this optimized model to efficiently denoise experimental data. To do so, we propose to solve the minimization of an objective function of the form

$$J_\varepsilon(\boldsymbol{u}) := \|\boldsymbol{u} - \boldsymbol{u}_{\text{data}}\|^2_{L^2(\Omega)} + \varepsilon \int_\Omega |\,\mathbf{div}(\boldsymbol{T} : \mathcal{E}(\boldsymbol{u}))|.$$

An efficient algorithm for this non-smooth optimization problem must to be developed. The obtained method will be tested on experimental data (for validation) and on *in-vivo* data from breast tumor detections. This work will be done in close collaboration with experimental scientists from CREATIS.

Candidate profile. We are looking for an enthusiastic candidate with a strong background in applied mathematics, and numerical analysis. Strong abilities in computer sciences will be appreciated. The applicant must enrolled in a research Master program. The following skills will be acquired during the internship, although prior knowledge on these topics are appreciated: inverse problems, non smooth/sparse optimisation, continuum mechanics, linear elasticity, galerkin approximation, finite element method, image processing, denoising and deep learning.

Practical information. The internship will take place in the ICJ laboratory (UMR 5208) within the MMCS team, at the INSA Lyon campus. The intern will be also fully integrated in the team of ANR PRC REWARD which will fund the internship ($\approx$ 600 euros/month). A continuation of this work as a PhD at Ecole Centrale de Lyon (MI-ICJ) is possible.

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